NOTATION

λ, specimen thermal conductivity; T, temperature; R_t, thermal resistance of constriction; a, specimen radius; h, specimen height; r, radius of area of heat introduction; ΔT, temperature difference; Q, power introduced into specimen.

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EFFECT OF A PHASE TRANSITION ON HEAT AND MASS TRANSFER IN DISPERSED FLOWS

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UDC 536.244:532.546.2

New effects are discussed which are characteristic for steady heat and mass transfer in a dispersed medium when a phase change occurs at particle surfaces.

Transport processes in real dispersed media and flows encountered in power engineering, chemical engineering, and other applications are frequently accompanied by homogeneous and heterogeneous chemical transformations and phase transitions. Examples are flows of hot liquids or unheated vapor — gas mixtures containing dispersed elements of a different phase, flows of reacting mixtures in the presence of catalyst particles, drying of granular materials, certain granulation processes, etc.

Reactions and phase transitions are significantly affected by the average temperature and concentration gradients of admixtures and reagents, and by average heat and mass fluxes, which in principle can lead to a change in the relations connecting the indicated thermodynamic forces and fluxes as compared with the relations characteristic of dispersed media with inert particles. Therefore one should expect both that the coefficients in these relations will differ from those for a homogeneous heat- and mass-transfer medium by amounts depending on the nature and intensity of the phase transitions and chemical transformations occurring and that specific cross effects, such as the effect of thermal diffusion, will appear.

As far as we know, these effects have not been specially studied. Using the general theory from [1] we show their presence by an example of a very simple problem of steady heat and mass transfer of vapor in a system with evaporating particles in the limit when the kinetics of evaporation (sublimation) is limited by the rate at which heat is supplied to the surface and can be described approximately in the temperature range of interest by a linear function

$$Q_\tau(T) = -LQ_c(T), \quad Q_c(T) = \alpha_1 + \alpha_2 T. \tag{1}$$

In addition, in order to concentrate our attention on the fundamental aspects of the matter and to simplify the calculations as much as possible, we make the same assumptions as

in studying the effects of unsteadiness in [2]. We consider media of low or moderate densities containing identical spherical particles and assume that the Peclet numbers, which characterize convective diffusion and heat conduction at the single-particle level, are small in comparison with unity. These assumptions enable us to neglect convective transfer connected with fluctuations of the phases and with their average relative motion, conductive heat transfer at contacts between particles, and also the effect of the nonoverlapping of particles [1, 2]. It is assumed that the particles are small in comparison with the linear scale of the temperature and concentration averages and that the latter is small in comparison with the scale of the macroscopic inhomogeneity of the medium. This enables us to consider the medium as a homogeneous continuum and to describe transfer processes by using the continuous methods of [1].

In the coordinate system fixed with respect to the average motion of the medium the equations of steady transfer can be written in the form [1]

$$-\nabla q_\tau + \psi_\tau = 0, \quad -\psi_\tau + h_\tau = 0,$$
$$-\Delta q_c + \psi_c = 0, \quad -\psi_c + h_c = 0,$$

(2)

in which the formal relations

$$q_\tau(R) = -\lambda_0 \nabla \tau(R) - (\lambda_1 - \lambda_0) n(R) \int\limits_{|R-R'| \leqslant a} \nabla \tau^*(R|R') \, dR',$$

$$q_c(R) = -D_0 \nabla c(R) - (D_1 - D_0) n(R) \int\limits_{|R-R'| \leqslant a} \nabla c^*(R|R') \, dR'$$

(3)

hold, and the integration is extended over the positions R' of the center of an isolated test particle in such a way that point R lies within it. Similar representations hold also for ψ_τ and ψ_c which characterize interphase exchange; they are not written down here, since in the case under consideration ψ_τ and ψ_c are the same as the functions h_τ and h_c which describe the strengths of the heat and vapor mass sources in the dispersed phase, which for a unit volume of the medium can be written as

$$h_\tau(R) = n(R) \int\limits_{r=a} Q_\tau(R+r) \, dr, \quad h_c(R) = n(R) \int\limits_{r=a} Q_c(R+r) \, dr.$$

(4)

Here the integration extends over the surface of the test particle.

The temperature and vapor density averages are expressed in terms of similar quantities for the phases of the medium in the form

$$\tau = \varepsilon \tau_0 + \rho \tau_1, \quad c = \varepsilon c_0 + \rho c_1, \quad \varepsilon = 1 - \rho.$$

(5)

It is clear that c_1 and, consequently, also c_0 are fictitious quantities with no physical meaning. They are introduced only to ensure a better analogy between heat- and mass-transfer processes and to determine the correct value of c in Eqs. (3) as $D_1 \to 0$. This example is actually used in [3] to calculate the effective diffusion coefficient in a granular medium.

Equations (2) determine the unknown fields τ_0, τ_1 and c_0, c_1. Using the fact that the standard transfer equations which hold in the individual phases are linear and using the general method of [1], all terms in (2) must be written as linear combinations of local values of these fields or their spatial derivatives with proper tensor dimensionality. The unknown coefficients in these combinations are determined a posteriori by using Eqs. (3), where the integrands in these formulas are found by solving special problems on the perturbations introduced into the average temperature and concentration fields by a fixed test particle.

Even after making these simplifying assumptions the calculations turn out to be very cumbersome. Therefore we use certain results which are generally not a priori obvious and are not verified by subsequent analysis. In the first place, as one should expect by analogy with [1-3], in the absence of contact transfer, which is treated independently in [4], the quantities τ_1 and c_1 are uniquely expressed in terms of τ_0 and c_0, so that in determining the form of the indicated combinations it is sufficient to take account only of the temperature and concentration fields in a continuous phase. Secondly, the component of the heat flux proportional to the concentration gradient is identically zero. Therefore we can take

$$q_\tau = -\lambda \nabla \tau_0, \quad q_c = -D \nabla c_0 - D_\tau \nabla \tau_0.$$

(6)

In addition, using (1)

$$h_\tau = -Lh_c, \quad h_c = \sigma_1 + \sigma_2\tau_0. \tag{7}$$

The coefficients λ, D, and D_τ in (6) and σ_1, σ_2 in (7) are presently unknown, and the purpose of the following analysis is to determine them explicitly.

Using (6) and (7) we obtain from (2)

$$\lambda\Delta\tau_0 - L(\sigma_1 + \sigma_2\tau_0) = 0,$$
$$D\Delta c_0 + D_\tau\Delta\tau_0 + \sigma_1 + \sigma_2\tau_0 = 0. \tag{8}$$

In the neighborhood of point R the functions τ_0 and c_0 can be written in the form

$$\tau_0(\mathbf{R} + \mathbf{r}) = T(\mathbf{R}) + \mathbf{E}(\mathbf{R})\,\mathbf{r} + \mathbf{r}\mathbf{M}(\mathbf{R})\,\mathbf{r} + \cdots,$$
$$c_0(\mathbf{R} + \mathbf{r}) = C(\mathbf{R}) + \mathbf{G}(\mathbf{R})\,\mathbf{r} + \mathbf{r}\mathbf{N}(\mathbf{R})\,\mathbf{r} + \cdots, \tag{9}$$

where each successive term in these expansions is of higher order in the ratio of the radius of the particle to the scale of the fields τ_0, c_0 than the preceding term. We have from (8) and (9)

$$\mathrm{Sp}\,\mathbf{M} = \frac{L(\sigma_1 + \sigma_2 T)}{2\lambda}, \quad \mathrm{Sp}\,\mathbf{N} = -\left(1 + \frac{LD_\tau}{\lambda}\right)\frac{\sigma_1 + \sigma_2 T}{2D}. \tag{10}$$

These same expansions can also be written in the neighborhood of any other point R'. The coefficients in these expansions, henceforth denoted by primes, are expressed as simple functions of the coefficients in (9) and the components of the vector R' − R.

We write the perturbed temperature and concentration fields outside the test particle in the form

$$\tau_0^* = \tau_0 + \tau', \quad c_0^* = c_0 + c'. \tag{11}$$

Using the same considerations in the formulation of the problem of the test particle as in [1-3], we obtain for τ' the temperature problem

$$\Delta\tau' - s^2\tau' = 0, \quad r > a; \quad \Delta\tau^* = 0, \quad r < a; \quad s^2 = \sigma_2 L/\lambda,$$
$$\tau' + \tau_0 = \tau^*, \quad \lambda\mathbf{n}\nabla(\tau' + \tau_0) = \lambda_1\mathbf{n}\nabla\tau^* - Q_\tau, \quad r = a; \tag{12}$$
$$\tau' \to 0, \quad r \to \infty; \quad \tau^* < \infty, \quad r = 0$$

and for c' the concentration problem

$$\Delta c' = -K\tau', \quad r > a; \quad \Delta c^* = 0, \quad r < a; \quad K = (s^2 D_\tau + \sigma_2)/D,$$
$$c' + c_0 = c^*, \quad D\mathbf{n}\nabla(c' + c_0) + D_\tau\mathbf{n}\nabla(\tau' + \tau_0) = D_1\mathbf{n}\nabla c^* - Q_c, \quad r = a; \tag{13}$$
$$c' \to 0, \quad r \to \infty; \quad c^* < \infty, \quad r = 0,$$

where the fields τ_0 and c_0 are defined in (9), and the fluxes Q_τ and Q_c in (1). The origin of the coordinate system is chosen at the center of the test sphere R'.

In solving problems (12) and (13) we use polynomials in spherical harmonics, taking account of the fact that terms proportional to spherical harmonics of order higher than the first do not contribute to the integrals in (3) and therefore need not be written down. We write the solution of problem (12) in the form

$$\tau' = A_0\left(\frac{a}{r}\right)^{1/2}K_{1/2}(sr) + A_1\mathbf{E}'\mathbf{r}\left(\frac{a}{r}\right)^{3/2}K_{3/2}(sr) + \cdots;$$
$$\tau^* = T' + A_0K_{1/2} + 1/3a^2\,\mathrm{Sp}\,\mathbf{M}' + (1 + A_1K_{3/2})\mathbf{E}'\mathbf{r} + \cdots, \tag{14}$$
$$K_j \equiv K_j(sa),$$

where the $K_j(x)$ are Macdonald functions, and the integration constants are

$$A_0 = -\frac{(\alpha_1 + \alpha_2 T')aL - 1/3a^2(2\lambda - \alpha_2 aL)\,\mathrm{Sp}\,\mathbf{M}'}{\lambda F_{1/2} + \alpha_2 aLK_{1/2}},$$

$$A_1 = -\frac{\lambda_1 - \lambda + \alpha_2 aL}{(\lambda_1 - \lambda + \alpha_2 aL)K_{3/2} + \lambda F_{3/2}}, \quad F_j = -a\frac{d}{dr}\left[\left(\frac{a}{r}\right)^j K_j(sr)\right], \quad r = a. \tag{15}$$

Since an analysis of solution (14) is very complex for the general case, we restrict ourselves to small α_1 and α_2 so that $\omega \sim (sa)^2 \ll 1$. Using (8) we note that the last inequality is equivalent to the assumption that the scale of τ_0, c_0 is very much larger than a and

is a necessary condition for the applicability of continuous methods. Retaining only first order terms in ω, using for Q_c the representation from (1) with $T = \tau^*$, and expressing T' and E' in terms of T, E, and the vector $\mathbf{r} = \mathbf{R} - \mathbf{R}'$, we evaluate the second integral in (4) and compare the result with the second of Eqs. (7). This leads to the equations

$$\sigma_i = \frac{3\rho}{a} \frac{\alpha_i F_{1/2}}{(1 - 1/2\omega\rho) F_{1/2} + \omega\varepsilon K_{1/2}}, \quad \omega = \frac{\alpha_2 aL}{\lambda}, \quad (16)$$

determining σ_1 and σ_2 as functions of known quantities and the single unknown parameter λ which we find by first calculating the average temperature of the dispersed phase. Averaging τ^* from (14) and (15) over the volume of the test particle, we have, similar to the preceding,

$$\tau_1 = \frac{\tau_0 F_{1/2} - (\alpha_1/\alpha_2)(\omega\varepsilon K_{1/2} - 1/2\omega\rho F_{1/2})}{(1 - 1/2\omega\rho) F_{1/2} + \omega\varepsilon K_{1/2}}. \quad (17)$$

By evaluating the integral in the first of Eqs. (3), using (5) and (17), and comparing the result with the first of Eqs. (6), we obtain a transcendental equation for the effective thermal conductivity:

$$\beta = \frac{[\varepsilon(1 - 1/2\omega\rho) + \rho] F_{1/2} + \omega\varepsilon^2 K_{1/2}}{(1 - 1/2\omega\rho) F_{1/2} + \omega\varepsilon K_{1/2}} + \frac{(\varkappa - 1)\beta\rho F_{3/2}}{(\varkappa - \beta + \omega) K_{3/2} + \beta F_{3/2}}, \quad \beta = \frac{\lambda}{\lambda_0}, \quad \varkappa = \frac{\lambda_1}{\lambda_0}. \quad (18)$$

Using the expansions of the Macdonald functions in powers of their arguments, we find from (16)-(18)

$$\sigma_i \approx \frac{3\rho}{a} \alpha_i, \quad \tau_1 \approx \tau_0 - \omega \left(1 - \frac{3}{2}\rho \right) \left(\tau_0 + \frac{\alpha_1}{\alpha_2} \right),$$

$$\beta \approx 1 - \omega \left(1 - \frac{3}{2}\rho \right)\rho + \frac{3\rho\beta(\varkappa - 1)}{\varkappa + 2\beta} \left\{ 1 - \frac{\omega}{\varkappa + 2\beta} \left[1 - \frac{\rho}{3}(\varkappa - \beta) \right] \right\}. \quad (19)$$

In particular, β can be written as $\beta^\circ + \delta\beta$, where

$$\beta^\circ = 1 + \frac{3(\varkappa - 1)\beta^\circ}{\varkappa + 2\beta^\circ}\rho, \quad \beta^\circ = \frac{\lambda^\circ}{\lambda_0}, \quad \omega^\circ = \frac{\alpha_2 aL}{\lambda^\circ},$$

$$\delta\beta \approx -\omega^\circ\rho \left\{ 1 - \frac{3}{2}\rho + \frac{3(\varkappa - 1)\beta^\circ}{(\varkappa + 2\beta^\circ)^2} \left[1 - \frac{\rho}{3}(\varkappa - \beta^\circ) \right] \right\}. \quad (20)$$

The equation for β° agrees with that obtained earlier in [5] for moderately concentrated dispersed media with inert particles. In the approximation of nonoverlapping particles the difference in the formulations of the test-particle problem in [3] and [5] disappears, and $\delta\beta$ describes the change in the effective thermal conductivity resulting from a phase transition. For values of $\varkappa$ not too much larger than unity, $\delta\beta < 0$; i.e., the phase transition worsens the heat transfer; in a medium with heat-conducting particles ($\varkappa \gg \rho^{-1}$), $\delta\beta$ is generally positive and heat transfer is improved.

The solution of problem (13) with the function τ' from (14) has the form

$$c' = \left[B_0 + kA_0 \int_1^{r/a} \sqrt{t}\, K_{1/2}(sat)\, dt \right] \frac{a}{r} + \left[\frac{1}{2} \mathbf{G}'\mathbf{r} + \left(B_1 + kA_1 \int_1^{r/a} t^{3/2} K_{1/2}(sat)\, dt \right)\mathbf{E}'\mathbf{r} \right] \left(\frac{a}{r} \right)^3 + \cdots;$$

$$c^* = C' + B_0 + \frac{1}{3} a^2 \mathrm{Sp}\,\mathbf{N}' + \frac{3}{2} \mathbf{G}'\mathbf{r} + B_1\mathbf{E}'\mathbf{r}, \quad k = \frac{aK}{s}, \quad (21)$$

where we obtain for the integration constants as $D_1 \to 0$

$$B_0 = \frac{a(\alpha_1 + \alpha_2 T')}{D} + A_0 \left[\left(k + \frac{\alpha_2 a}{D} \right) K_{1/2} - \frac{D_\tau}{D} F_{1/2} \right] + \frac{1}{3} a^2 \left(2\mathrm{Sp}\,\mathbf{N}' + \frac{2D_\tau + \alpha_2 a}{D}\mathrm{Sp}\,\mathbf{M}' \right),$$

$$B_1 = \frac{1}{2} \frac{D_\tau + \alpha_2 a}{D} + \frac{1}{2} A_1 \left(kK_{1/2} + \frac{D_\tau + \alpha_2 a}{D} K_{3/2} - \frac{D_\tau}{D} F_{3/2} \right). \quad (22)$$

The integrals in (21) can be expressed in terms of exponential functions and also all the quantities in (21) and (22) except D and D_τ, which are determined by the above relations and can be considered known.

To determine the effective coefficients of diffusion and thermal diffusion we compare the expressions for q_c from (3) and (6). Equations (21) and (22) enable us to evaluate the integral in (3) and to find a formal expression for c_1 necessary to estimate the value of c appearing in (3) in accord with (5). As a result the flux q_c is written as a sum of terms proportional to G and E from (9), with coefficients of proportionality depending on D and D_T as parameters. Equating these coefficients to the corresponding coefficients in (6) we obtain two algebraic equations for the latter parameters. The solution of these equations completes the investigation. The calculations and final expressions for these parameters are too complicated to present here. We write down only approximate relations which are accurate to first-order terms in α_1 or α_2, i.e., in ω.

Using (10), (15), and (19) we obtain from (21) at point R, to the accuracy indicated

$$c_1 \approx c_0 + (1 - 3 \cdot 2\rho)(\alpha_1 + \alpha_2\tau_0)(a/D),$$

$$\nabla c \approx \mathbf{G} + (1 - 3 \cdot 2\rho)\rho\,(\alpha_2 a/D)\,\mathbf{E}.$$

$$(23)$$

The evaluation of the integral for q_c in (3) gives

$$n \int_v \nabla c^* d\mathbf{R}' \approx \rho \left(\frac{3}{2}\mathbf{G} + B_1\mathbf{E} \right),$$

$$(24)$$

so that by comparing (3) and (6) we obtain after a simple calculation

$$D = D^\circ = (1 - 3/2\rho)\,D,$$

$$D_\tau = \alpha_2 a\rho \, \frac{2\varkappa + (1 - 9\rho)\,\beta^\circ}{2\varkappa + 2(2 - 3\rho)\,\beta^\circ},$$

$$(25)$$

where β° is defined in (20). It is easy to verify that the first of Eqs. (25), which shows that the effective coefficient of diffusion in the case under discussion does not depend on the existence of a phase transition, is valid in any approximation in α_1 and α_2 and agrees with the result obtained earlier in [5]. The second of Eqs. (25) shows that in a dispersed medium with evaporating particles an effect is observed which is analogous to the well-known phenomenon of thermal diffusion in gases and determines the corresponding coefficient of thermal diffusion.

The fact that D is independent of a phase transition and the absence of diffusive heat conduction, i.e., the component of the heat flux proportional to the gradient of the vapor concentration, are due entirely to the simplifying assumption made in writing (1) that the kinetics of evaporation does not depend on the vapor concentration, as is possible if the partial pressure of the vapor is very much smaller than the saturated vapor pressure. In the more general case a phase transition will affect both λ and D, and two overlapping effects will appear — thermal diffusion and diffusive heat conduction. It is clear that both effects can significantly influence the intensity of transfer processes in regions with large gradients, for example, close to walls.

Similar phenomena will occur for chemical transformations as well as for phase transitions. Thus, for example, all the results presented automatically carry over to dispersed media with a catalytic reaction at particle surfaces if we understand by τ and c the concentrations of a reagent and a reaction product, respectively, and the kinetics of the reaction is described by equations of the type (1), where L plays the role of a stoichiometric coefficient. Generalization to the case of several chemical transformations or phase transitions with kinetics depending on temperature, vapor concentrations, and the substances taking part in the reactions is simple in principle, but involves cumbersome calculations.

NOTATION

A_i, B_i, integration constants; a, radius of particles; c, concentration of vapor; D, coefficient of diffusion; D_τ, coefficient of thermal diffusion; E, temperature gradient; G, concentration gradient; h_τ, h_c, strengths of heat and vapor mass sources per unit volume of medium; K, k, coefficients defined in (13) and (21); L, heat of vaporization; M, N, tensors introduced in (9); n, number density of particles; n, unit vector along outward normal; Q_τ, Q_c, strengths of heat and vapor mass sources per unit area of phase interface; q_τ, q_c, average heat and vapor mass fluxes; s, parameter introduced in (12); α_i, coefficients in (1); $\delta\beta$, correction to β° defined in (20); ε, porosity; λ, thermal conductivity; ρ, volume concentration of particles; σ_i, coefficients introduced in (7); τ, temperature; ψ_τ, ψ_c, interphase

heat and vapor mass fluxes per unit volume of medium; ω, parameter defined in (16). Indices: 0, 1, continuous and dispersed phases, respectively; *, quantities perturbed by test particle; °, parameters of medium when there is no phase transition.

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RATE OF BUBBLE RISE IN A NONUNIFORM FLUIDIZED BED

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UDC 532.546

The dimensions and rate of rise of bubbles in a column 700 mm in diameter were determined experimentally. On the basis of a two-phase model, a calculated correlation was obtained which related the relative velocity of a gas bubble to its diameter.

At the present time, the problem of the rise of a single artificial bubble in a uniform fluidized bed has been studied in some detail in the literature and the dependence of its velocity on diameter obtained [3]:

$$v_{\text{B}} = k \sqrt{g \, \frac{1}{2} \, D_{\text{B}}}, \qquad (1)$$

where $0.8 \leq k \leq 1.2$.

In a nonuniform fluidized bed, coalescence of bubbles occurs and there is only fragmentary data on the rates of rise and dimensions of gaseous nonuniformities averaged across the fluidized bed [4-6, 9, 10]. There is almost no data for columns of large diameter [11]. At the same time, this information is of great interest for the design of commercial equipment.

The present paper is aimed at the problems of obtaining information about the quantities v_B and D_B in a column 700 mm in diameter and of unifying the known experimental data.

The following correlation for the vertical size of a bubble was obtained [8] for equipment of different sizes (100 mm $\leq$ D $\leq$ 700 mm):

$$D_h = \frac{1.3}{g^{1/3}} \, [(u - u_0) \, h]^{2/3}, \qquad (2)$$

and is connected to bubble diameter through the relation [13] $D_h = 0.7 D_B$. Equation (2) is valid for small particles (70 < d < 400 μ) and uniform gas distribution.

For similar conditions a relationship was found [2] for expansion of the bed:

$$p - 1 = 0.7 \left(\frac{H_0}{D} \right)^{1/2} \mathrm{Fr}^{1/3} \qquad (3)$$

for D = 100-700 mm and 0.5 < H_0/D < 2.